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# Applying Vectorization to Lattice QCD Calculations

Shun Xu, Zhong Jin

Computer Network Information Center  
Chinese Academy of Sciences

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- **Introduction to lattice QCD**
  - Background in lattice QCD
  - The computational method and its challenge
- **Vectorization optimization of lattice QCD**
  - The principle of algorithm
  - On Intel & Sunway processors
- **Summary**

**Quantum ChromoDynamics (QCD)** : a fundamental theory of strong interaction

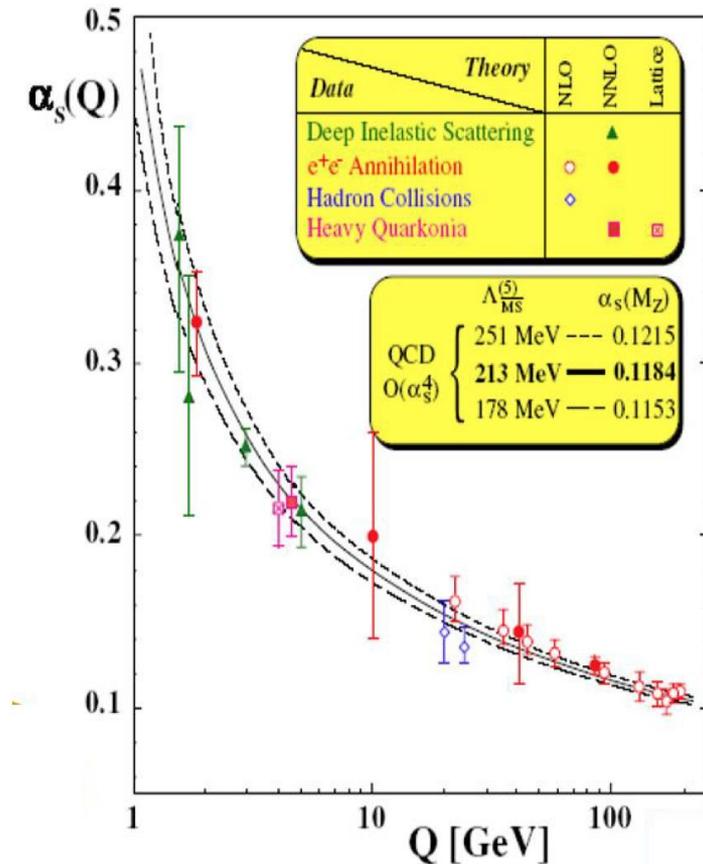
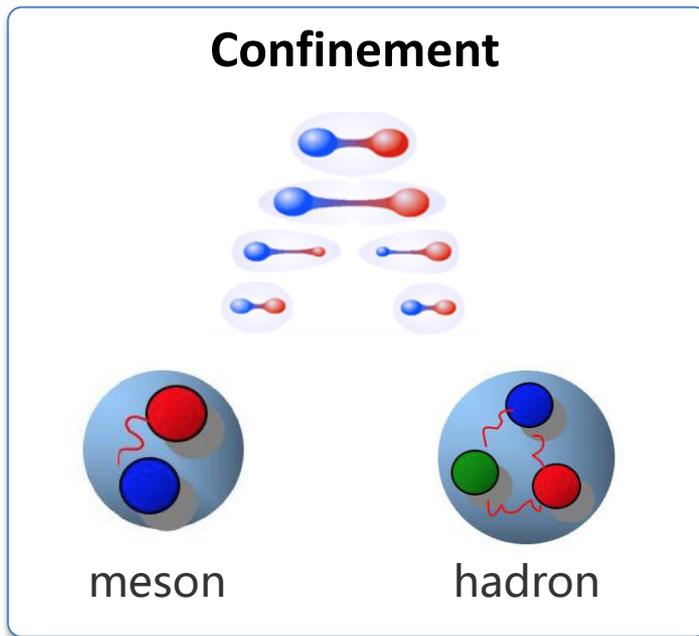


photo PRB



photo PRB  
H. David Politzer

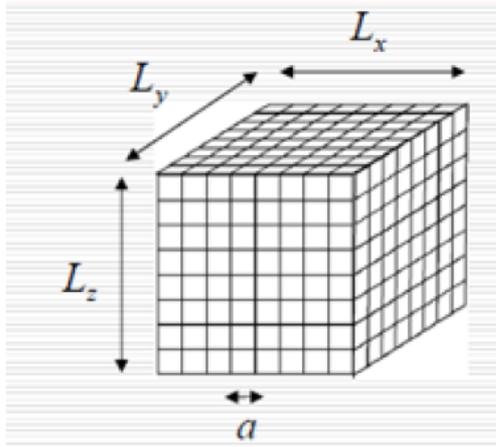


photo PRB  
Frank Wilczek



For the discovery of **asymptotic freedom** in the theory of the **strong interaction**.

Unfortunately, the quark **confinement** problem can not be solved directly from the perturbative method of QCD, but the discrete and numerical solution on lattice is an effective measure, i.e. lattice QCD.



$$\begin{aligned} N &= L^3 \times T \\ \{P_l, A_l\}, l &= 8 \times 4 \times N; \\ \{\pi_i, \phi_i\}, i &= 12N; \\ \{\pi_i^*, \phi_i^*\}, i &= 12N \\ M[U] &: 12N \times 12N (\text{matrix}) \\ Q[U] &= M^+[U]M[U] \end{aligned}$$

$$\begin{aligned} H &= \frac{1}{2} \sum_l P_l^2 + \sum_i \pi_i^* \pi_i + S_G[U] + S_{PF}[U, \phi, \phi^*] \\ S_{PF} &= \sum_{i,j} \phi_i^* Q^{-1}_{ij} \phi_j = \phi^+ Q^{-1} \phi \\ \bar{O} &= \frac{1}{Z} \int [DUDP] \int D\phi D\phi^* D\pi D\pi^* e^{-H} O[U] \\ Z &= \int DUD\phi D\phi^* DPD\pi D\pi^* e^{-H} \end{aligned}$$

Space-time discretization

Huge degree of freedom

Path integral quantification: from quantum field theory to statistical ensemble

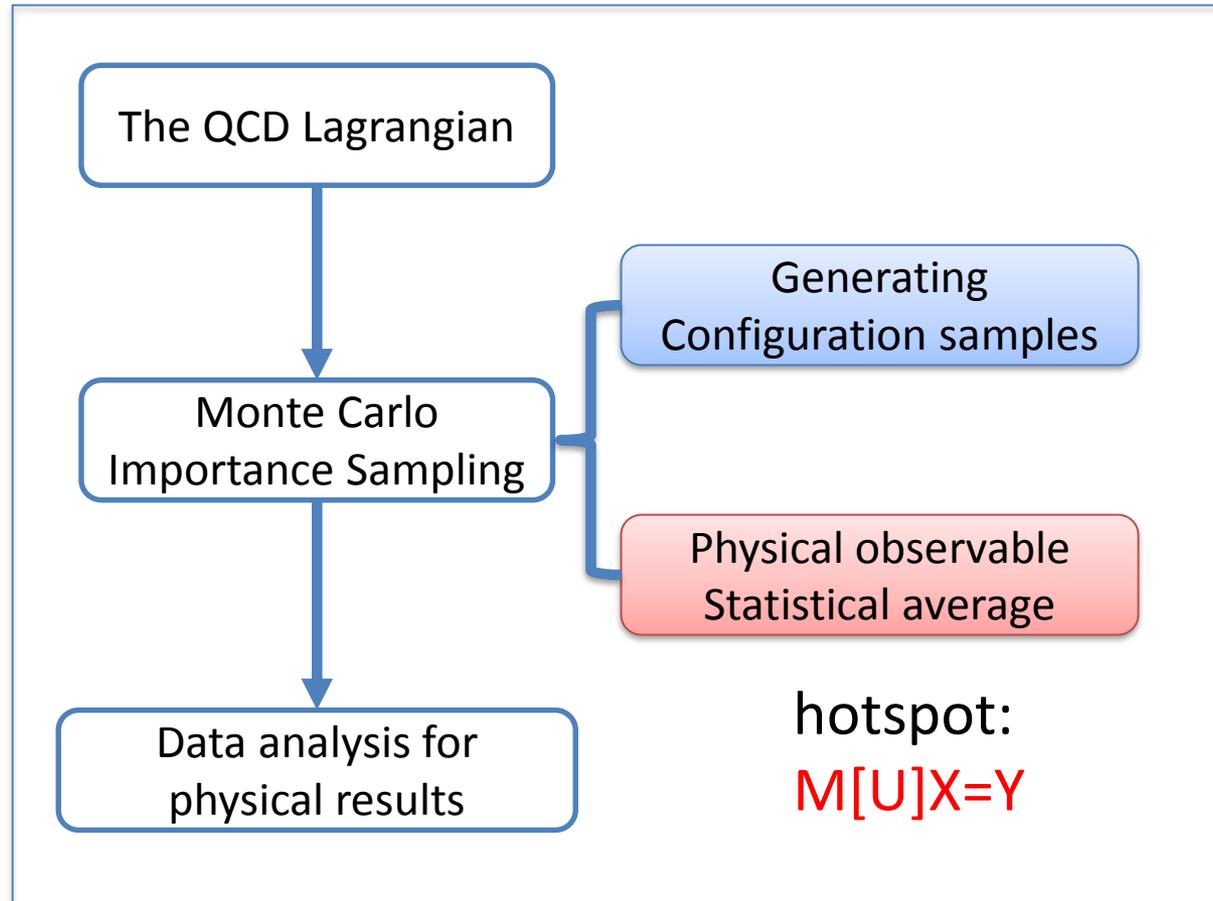
Lattice QCD: A first-principle method to study the QCD non-perturbative properties



- Lattice QCD is of great theoretical significance
  - the study of strong interaction
  - the accurate inspection of the standard model
  - the search of new physical results
- Lattice QCD numerical simulation is expensive computationally
  - Won **the Gordon Bell Prize** in 1988, 1998 and 2006 and the finalist in 2018.

# The Lattice QCD Method

The numerical computation method of lattice QCD



U: 3x3- dimensional matrix

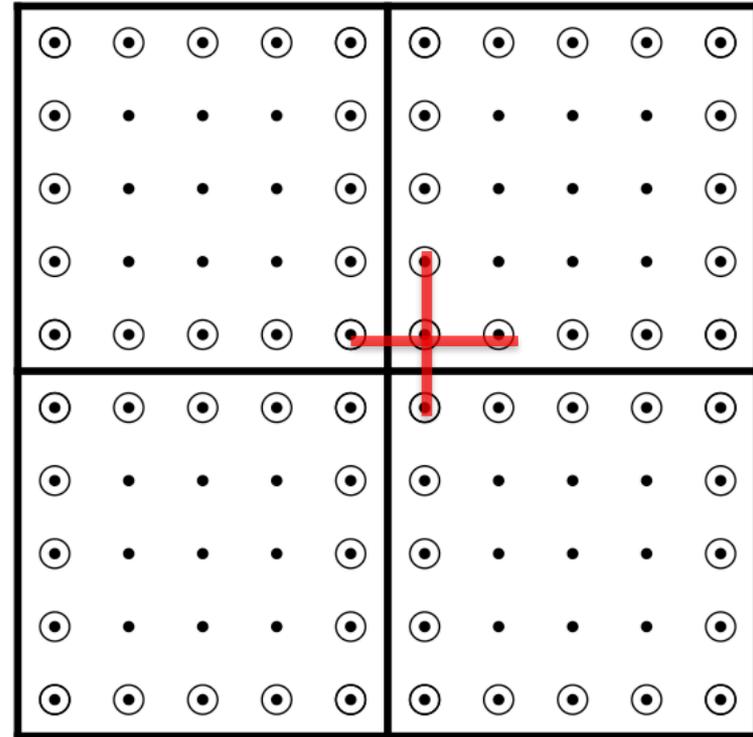
M[U]:  $L^3 \times T \times 12$ -dimensional sparse matrix

# The Computational Challenges

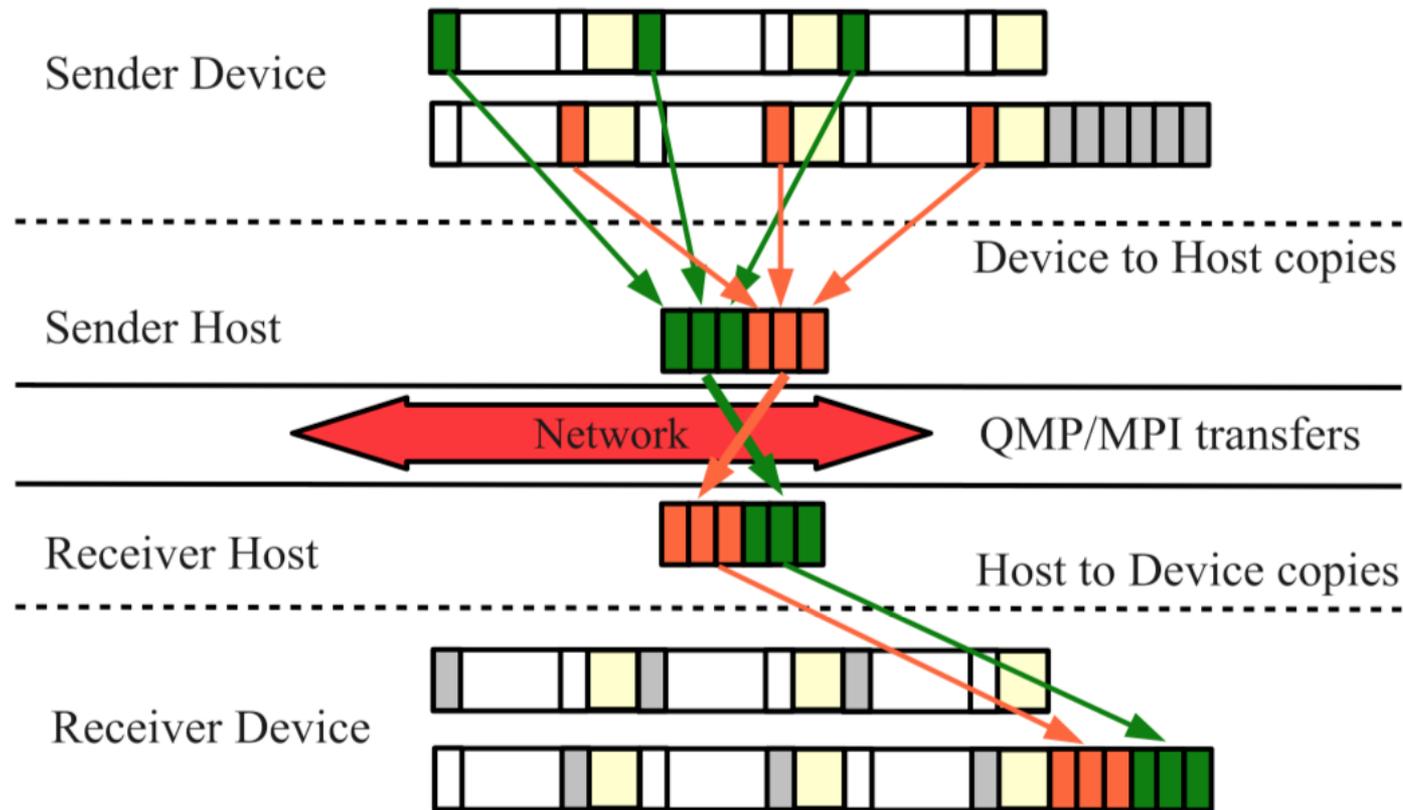
$$\begin{aligned} M_{x,x'} &= -\frac{1}{2} \sum_{\mu=1}^4 \left( P^{-\mu} \otimes U_x^\mu \delta_{x+\hat{\mu},x'} + P^{+\mu} \otimes U_{x-\hat{\mu}}^{\mu\dagger} \delta_{x-\hat{\mu},x'} \right) \\ &\quad + (4 + m + A_x) \delta_{x,x'} \\ &\equiv -\frac{1}{2} D_{x,x'} + (4 + m + A_x) \delta_{x,x'}. \end{aligned}$$

The computational challenges

1. a statistical problem for a multi-degree-of-freedom system in neighbor interaction.
2. a computation-intensive task
  - high parallelism
  - high scalability



1. **Task:** Overlap between calculation and communication
2. **Instruction:** Vectorization optimization of high-dimensional data



**Ref.:** R. Babich, M. Clark, B. Joo, SC10, arXiv:1011.0024

To cut the space-time of the periodic boundary condition into a four-dimensional lattice array

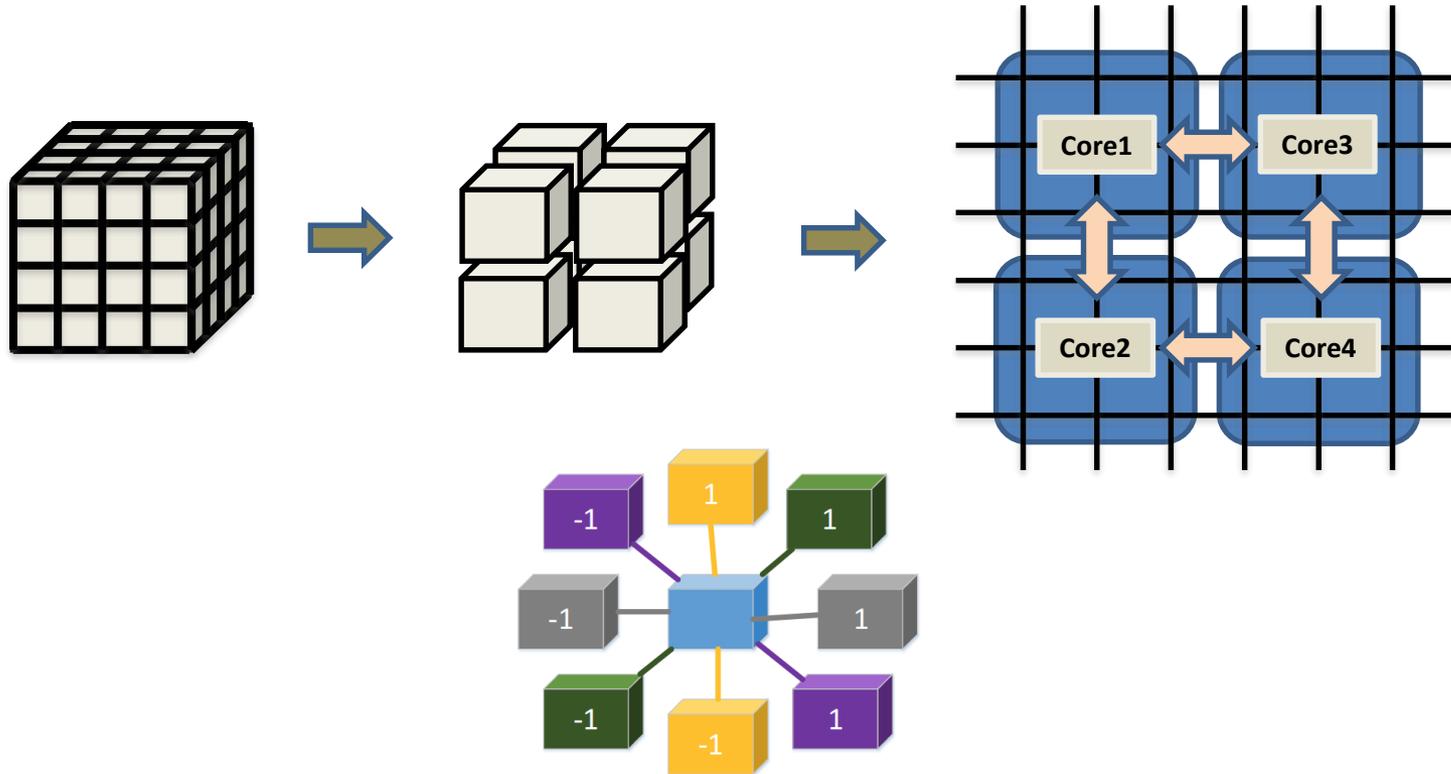
The fermion field quantity representing quark is placed at each lattice site, which is the Gell-Mann number with three chromatic components and four spinor components (12 complex vectors).

$$\phi(x, y, z, t) = \left( \begin{array}{c} \left( \begin{array}{c} d_{11} \\ d_{12} \\ d_{13} \end{array} \right) \\ \left( \begin{array}{c} d_{21} \\ d_{22} \\ d_{23} \end{array} \right) \\ \left( \begin{array}{c} d_{31} \\ d_{32} \\ d_{33} \end{array} \right) \\ \left( \begin{array}{c} d_{41} \\ d_{42} \\ d_{43} \end{array} \right) \end{array} \right)$$

The gauge field of gluon is placed on the connection between adjacent sites, which can be written as a 3x3 complex matrix with unitary mode on color space. Each lattice has eight such matrices, each connected to eight neighbors.

$$U_{\mu}(x, y, z, t) = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}$$

# Data parallelism in different levels



4D stencil computation in Lattice QCD

$$\begin{pmatrix} a + bi, & c + di, & e + fi \\ g + hi, & j + ki, & l + mi \\ n + pi, & q + ri, & s + ti \end{pmatrix} \begin{pmatrix} u + vi \\ w + xi \\ y + zi \end{pmatrix} = \begin{pmatrix} (au + cw + ey - bv - dx - fz) + (av + cx + ez + bu + dw + fy)i \\ (gu + jw + ly - hv - kx - mz) + (gv + jx + lz + hu + kw + my)i \\ (nu + qw + sy - pv - rx - tz) + (nv + qx + sz + pu + rw + ty)i \end{pmatrix}$$

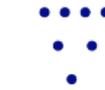
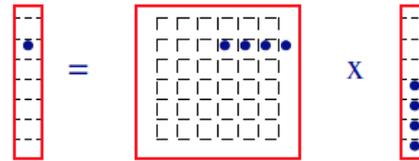
- Programming of vectorization
  - SSE, AVX, AVX2 and AVX512
  - ASM and Intrinsic function
  - OpenMP simd directive
- Packages available
  - QDP++
    - Scalarsite sse library (for blas and linalg)
  - MILC
    - Single precision SSE routines for MILC
  - **Grid**
    - C++ 11 template classes for SIMD vectors

# SIMD in Grid

## A SIMD vector parallelism framework offered by Grid package

1. The abstraction of vector operations using modern C++ 11
2. SSE, AVX, AVX2, FMA4, IMCI and AVX512

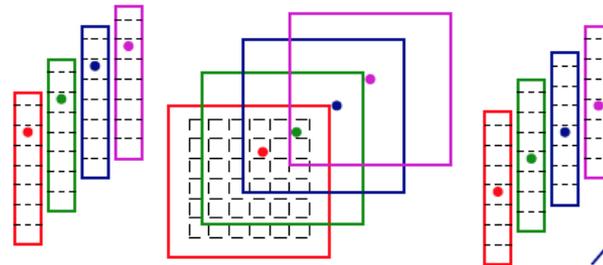
Vector = Matrix x Vector



Reduction of vector sum is bottleneck for small N



Many vectors = many matrices x many vectors

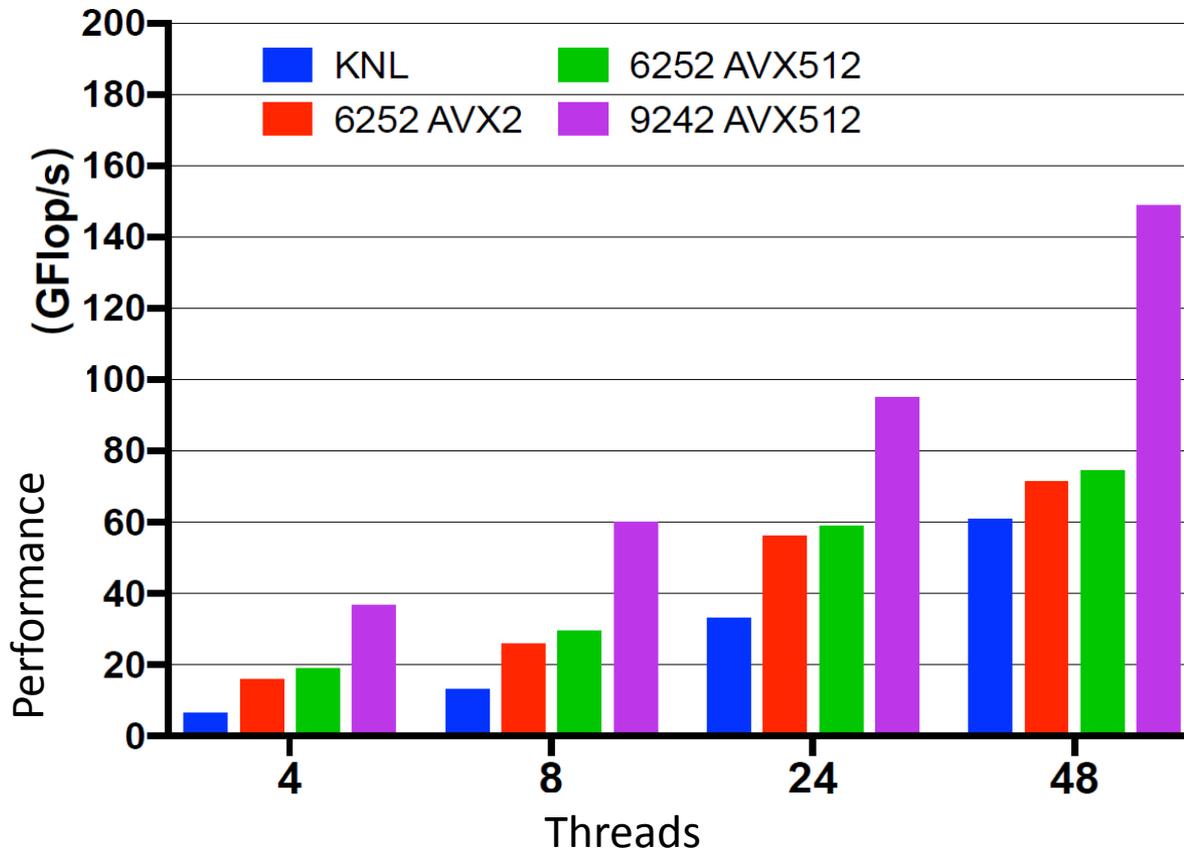


No reduction or SIMD lane crossing operations.

```
#if defined (SSE2)
    typedef __m128 zvec;
#endif
#if defined (AVX1) || defined (AVX2)
    typedef __m256 zvec;
#endif
#if defined (AVX512)
    typedef __m512 zvec;
#endif
class vComplexD {
    zvec v;
    // Define arithmetic operators
    friend inline vComplexD operator + (vComplexD a, vComplexD b);
    friend inline vComplexD operator - (vComplexD a, vComplexD b);
    friend inline vComplexD operator * (vComplexD a, vComplexD b);
    friend inline vComplexD operator / (vComplexD a, vComplexD b);
    static int Nsimd(void);
}
```

**Ref.:** *Grid: A next generation data parallel C++ QCD library.* Boyle, Peter & Cossu, Guido & Yamaguchi, Azusa & Portelli, Antonin. (2016).

# Performance results of Grid



**KNL:** Intel Xeon Phi 7210  
**6252:** Intel Xeon Gold 6252  
**9242:** Intel Xeon Platinum 9242

Grid tests of vectorization on different processors

BEHIN

## SU(3) Vectorization multiplication

for i in  $(a_1, a_2, a_3)$

tmp1 = simd\_load(i,i,i)

for j in  $(\alpha, \gamma, \mu)$

tmp2 = simd\_load( $j_1, j_2, j_3, j_4$ )

result\_re += simd\_mul(tmp1,tmp2)

for j in  $(\beta, \delta, \nu)$

tmp2 = simd\_load( $j_1, j_2, j_3, j_4$ )

result\_im += simd\_mul(tmp1,tmp2)

for i in  $(b_1, b_2, b_3)$

for j in  $(\alpha, \gamma, \mu)$

tmp2 = simd\_load( $j_1, j_2, j_3, j_4$ )

result\_im += simd\_mul(tmp1,tmp2)

for j in  $(\beta, \delta, \nu)$

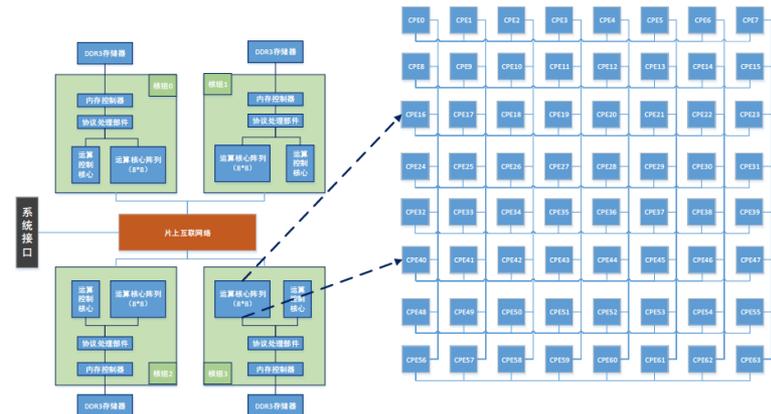
tmp2 = simd\_load( $j_1, j_2, j_3, j_4$ )

result\_re += simd\_mul(tmp1,tmp2)

END

### Algorithm of vectorization

1. Partition data by the Z and T axis.
2. 256-bit SIMD vectorization
3. Register communication

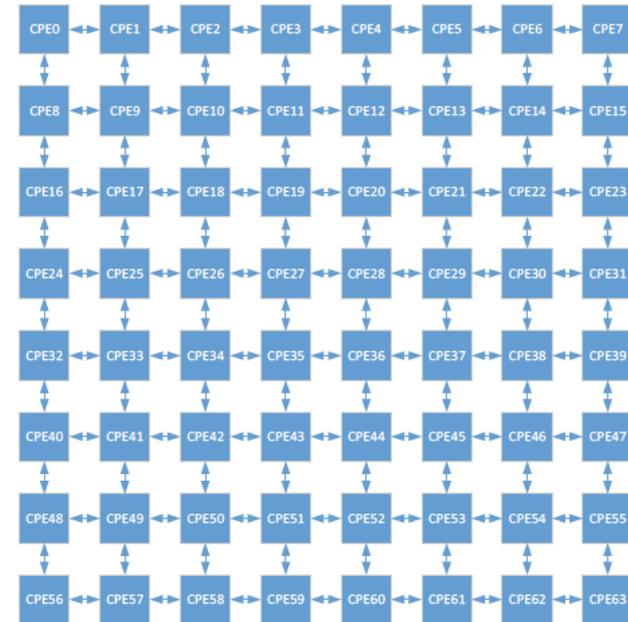
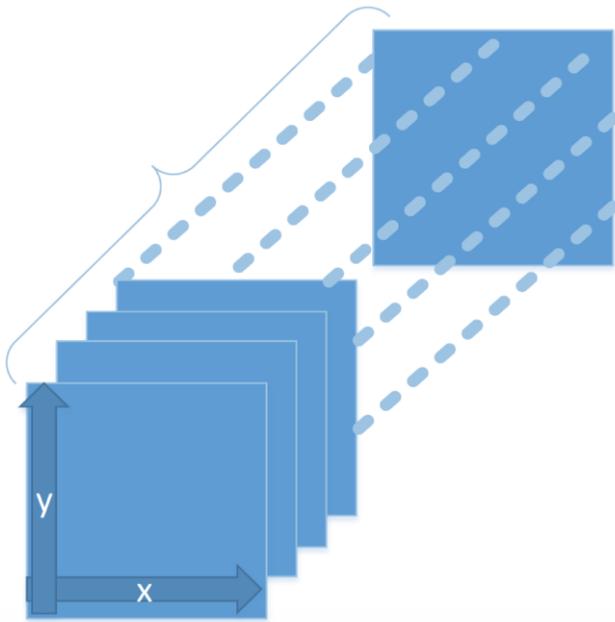


The architecture of Sunway many-core processor

[Sunway TaihuLight Supercomputer](#)

# Data organization for vectorization

To fit 64KB size of LDM(Local Data Memory) in computing processing element (CPE)

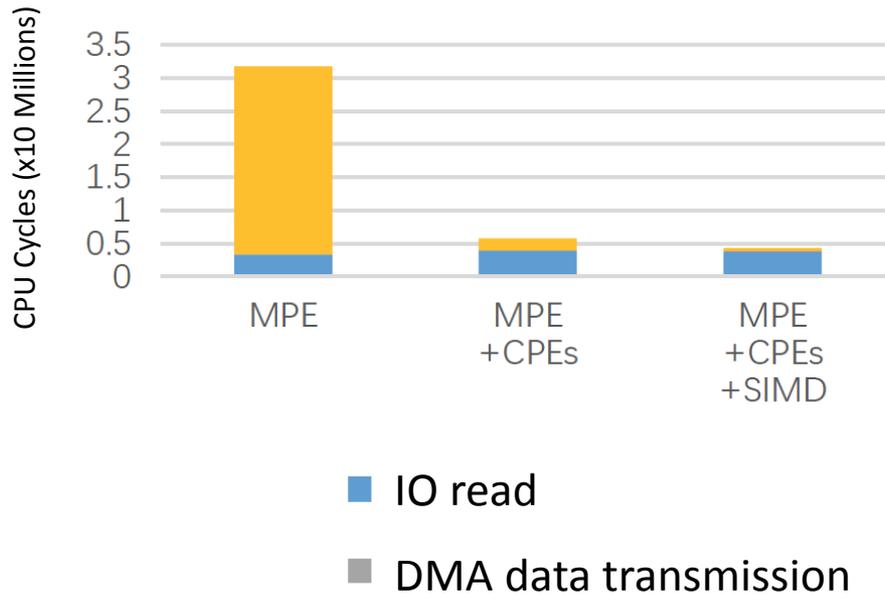


Data segmentation for CPE vectorization

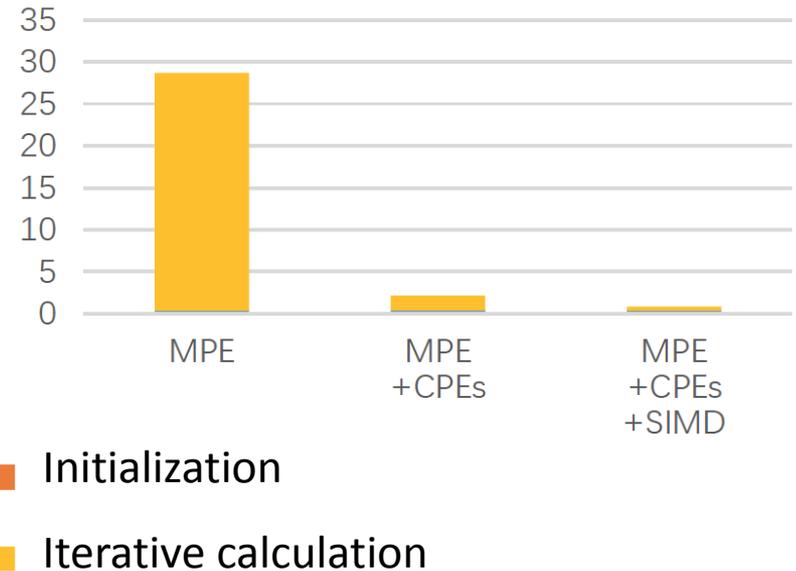
Register communication between CPEs

# Performance results

The running time ratio of each part in 10 times iterative calculation



The running time ratio of each part in 100 times iterative calculation



- We are suffering from
  - The poor portability of vectorization code
  - The complexity hardware details overexposed to user
- We are looking forward to
  - A unified programming model with data parallelism
  - Library available on diverse architectures (e.g. SSE, AVX, ASIMD, and NEON).
  - A uniform API for domainal applications

- Lattice QCD is a useful non-perturbative theoretical calculation method, by defining field variables at discrete time-space points.
- High-performance computational methods are required to optimize the data computation and communication in large-scale lattice QCD.
- **My view:** *A Unified data parallel programming model* (e.g. One API by Intel) is expected by lattice QCD application.

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xushun@sccas.cn