

IXPUG 2019 Annual Conference at CERN

Applying Vectorization to Lattice QCD Calculations

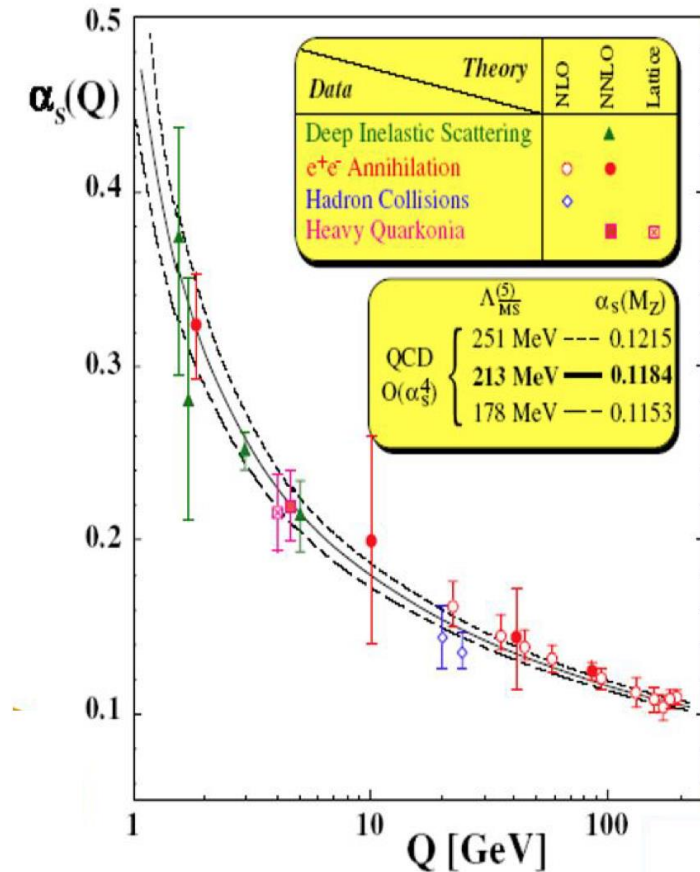
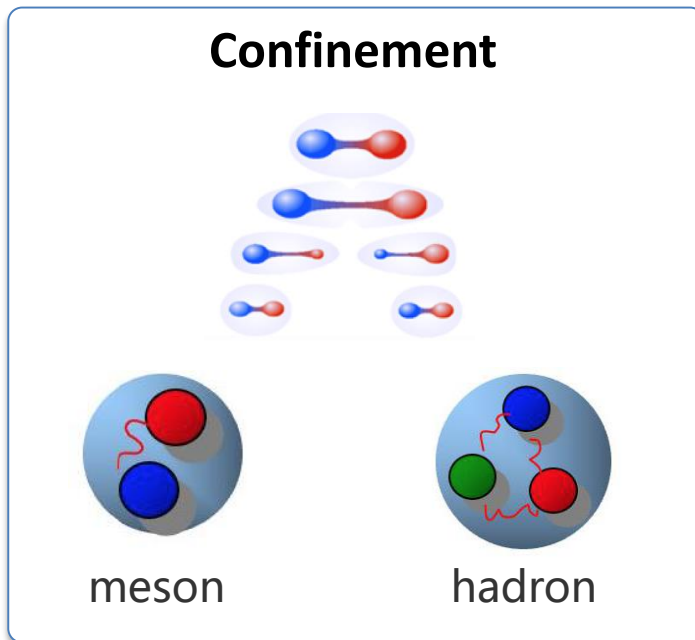
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- **Introduction to lattice QCD**
 - Background in lattice QCD
 - The computational method and its challenge
- **Vectorization optimization of lattice QCD**
 - The principle of algorithm
 - On Intel & Sunway processors
- **Summary**

Quantum ChromoDynamics (QCD) : a fundamental theory of strong interaction



David J. Gross



H. David Politzer

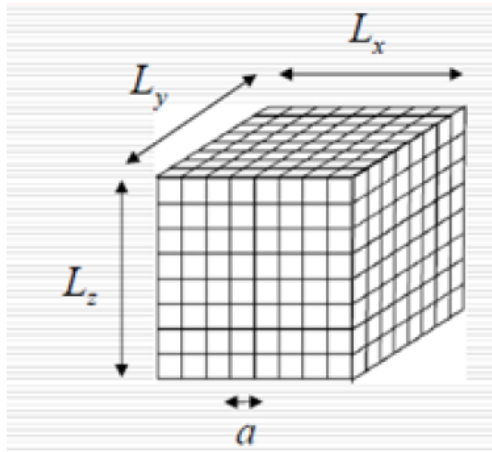


Frank Wilczek



For the discovery of **asymptotic freedom** in the theory of the **strong interaction**.

Unfortunately, the quark **confinement** problem can not be solved directly from the perturbative method of QCD, but the discrete and numerical solution on lattice is an effective measure, i.e. lattice QCD.



Space-time
discretization

$$\begin{aligned} N &= L^3 \times T \\ \{P_l, A_l\}, l &= 8 \times 4 \times N; \\ \{\pi_i, \phi_i\}, i &= 12N; \\ \{\pi_i^*, \phi_i^*\}, i &= 12N \\ M[U] &: 12N \times 12N (\text{matrix}) \\ Q[U] &= M^+[U]M[U] \end{aligned}$$

Huge degree
of freedom

$$\begin{aligned} H &= \frac{1}{2} \sum_l P_l^2 + \sum_i \pi_i^* \pi_i + S_G[U] + S_{PF}[U, \phi, \phi^*] \\ S_{PF} &= \sum_{i,j} \phi_i^* Q^{-1}_{ij} \phi_j = \phi^+ Q^{-1} \phi \\ \overline{O} &= \frac{1}{Z} \int [DUDP] \int D\phi D\phi^* D\pi D\pi^* e^{-H} O[U] \\ Z &= \int DUD\phi D\phi^* DPD\pi D\pi^* e^{-H} \end{aligned}$$

Path integral quantification: from
quantum field theory to statistical
ensemble

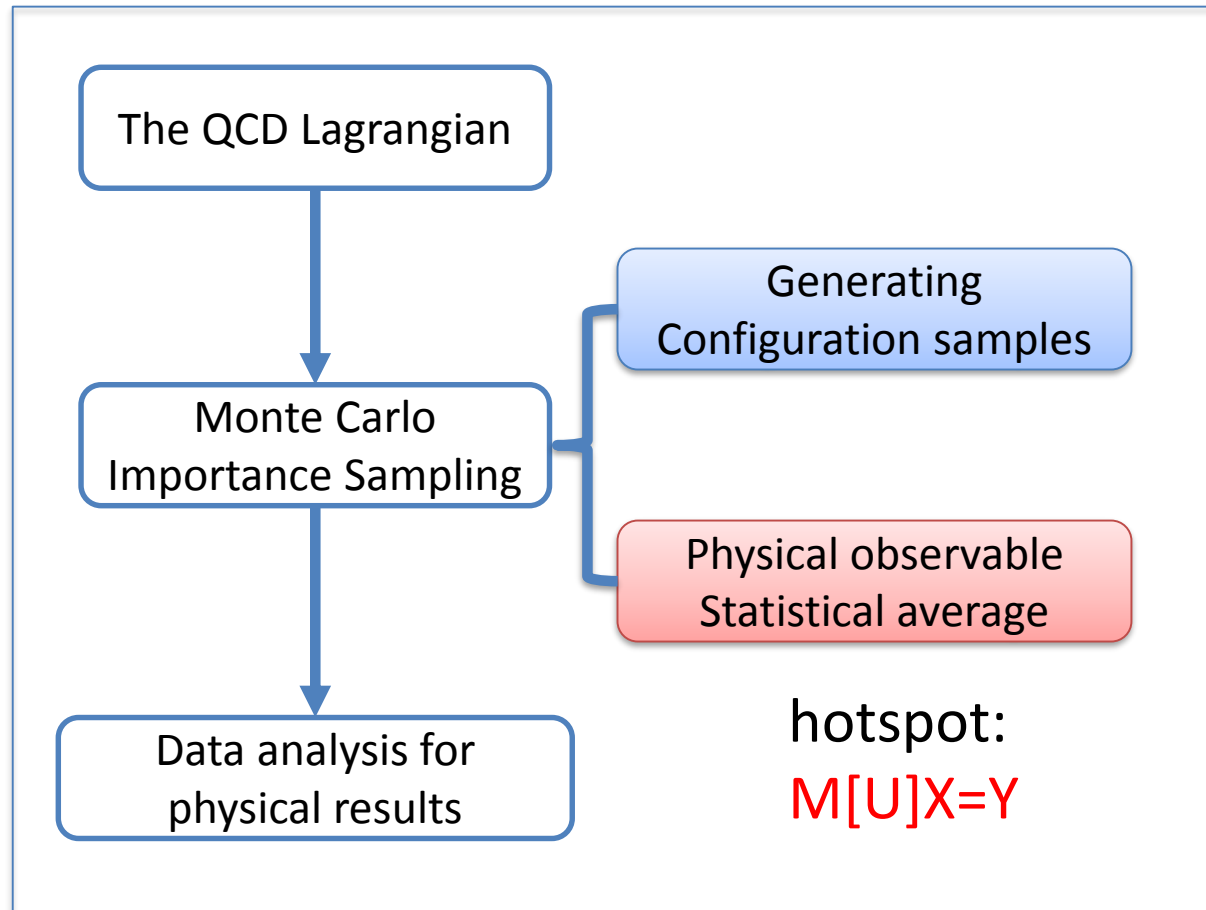
Lattice QCD: A first-principle method to study the QCD non-perturbative properties



- Lattice QCD is of great theoretical significance
 - the study of strong interaction
 - the accurate inspection of the standard model
 - the search of new physical results
- Lattice QCD numerical simulation is expensive computationally
 - Won **the Gordon Bell Prize** in 1988, 1998 and 2006 and the finalist in 2018.

The Lattice QCD Method

The numerical computation method of lattice QCD



U: 3x3- dimensional matrix

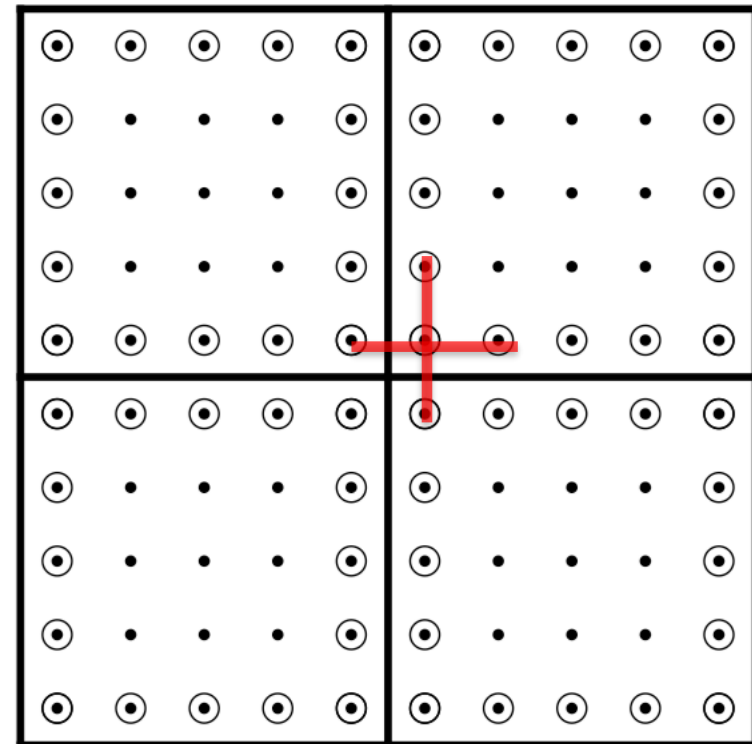
$M[U]$: $L^3 \times T \times 12$ -dimensional sparse matrix

The Computational Challenges

$$\begin{aligned}
 M_{x,x'} &= -\frac{1}{2} \sum_{\mu=1}^4 \left(P^{-\mu} \otimes U_x^{\mu} \delta_{x+\hat{\mu},x'} + P^{+\mu} \otimes U_{x-\hat{\mu}}^{\mu\dagger} \delta_{x-\hat{\mu},x'} \right) \\
 &\quad + (4 + m + A_x) \delta_{x,x'} \\
 &\equiv -\frac{1}{2} D_{x,x'} + (4 + m + A_x) \delta_{x,x'}.
 \end{aligned}$$

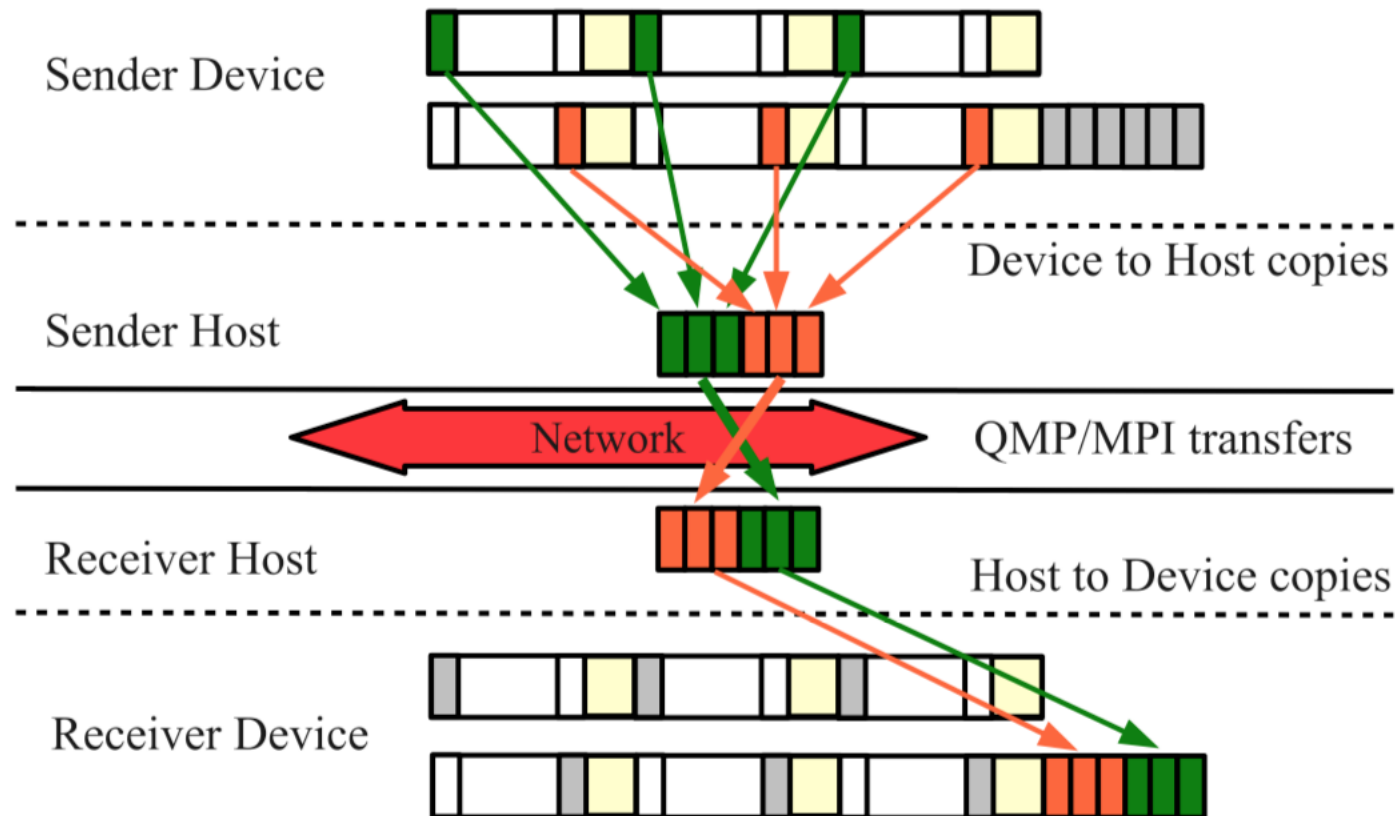
The computational challenges

1. a statistical problem for a multi-degree-of-freedom system in neighbor interaction.
2. a computation-intensive task
 - high parallelism
 - high scalability



Methods to improve performance

1. **Task:** Overlap between calculation and communication
2. **Instruction:** Vectorization optimization of high-dimensional data



Ref.: R. Babich, M. Clark, B. Joo, SC10, arXiv:1011.0024

To cut the space-time of the periodic boundary condition into a four-dimensional lattice array

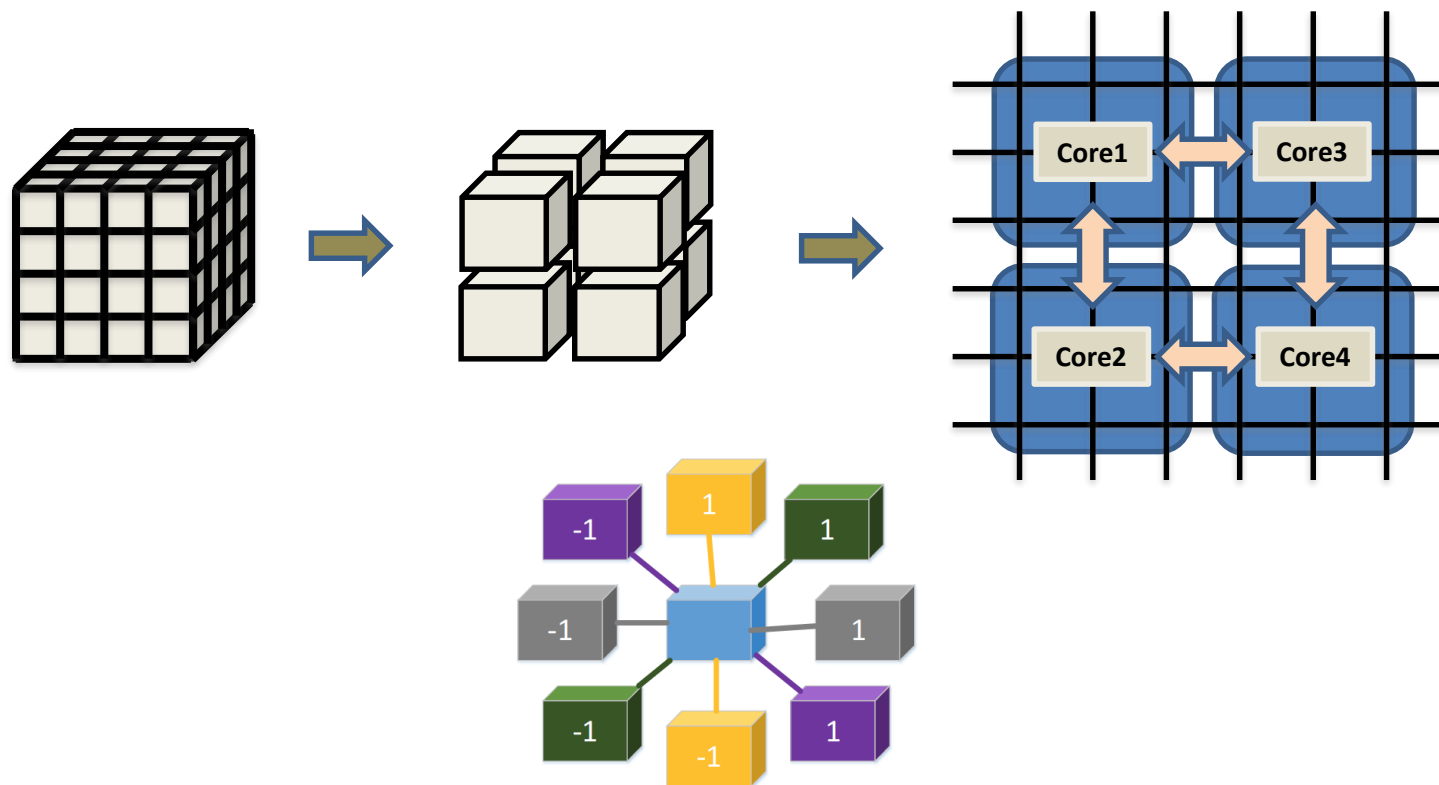
The fermion field quantity representing quark is placed at each lattice site, which is the Glasman number with three chromatic components and four spinor components (12 complex vectors).

$$\phi(x, y, z, t) = \begin{pmatrix} \begin{pmatrix} d_{11} \\ d_{12} \\ d_{13} \end{pmatrix} \\ \begin{pmatrix} d_{21} \\ d_{22} \\ d_{23} \end{pmatrix} \\ \begin{pmatrix} d_{31} \\ d_{32} \\ d_{33} \end{pmatrix} \\ \begin{pmatrix} d_{41} \\ d_{42} \\ d_{43} \end{pmatrix} \end{pmatrix}$$

The gauge field of gluon is placed on the connection between adjacent sites, which can be written as a 3x3 complex matrix with unitary mode on color space. Each lattice has eight such matrices, each connected to eight neighbors.

$$U_{\mu}(x, y, z, t) = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}$$

Data parallelism in different levels



4D stencil computation in Lattice QCD

$$\begin{pmatrix} a + bi, & c + di, & e + fi \\ g + hi, & j + ki, & l + mi \\ n + pi, & q + ri, & s + ti \end{pmatrix} \begin{pmatrix} u + vi \\ w + xi \\ y + zi \end{pmatrix} = \begin{pmatrix} (au + cw + ey - bv - dx - fz) + (av + cx + ez + bu + dw + fy)i \\ (gu + jw + ly - hv - kx - mz) + (gv + jx + lz + hu + kw + my)i \\ (nu + qw + sy - pv - rx - tz) + (nv + qx + sz + pu + rw + ty)i \end{pmatrix}$$

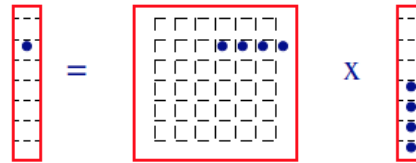


- Programming of vectorization
 - SSE, AVX, AVX2 and AVX512
 - ASM and Intrinsic function
 - OpenMP simd directive
- Packages available
 - QDP++
 - Scalarsite sse library (for blas and linalg)
 - MILC
 - Single precision SSE routines for MILC
 - **Grid**
 - C++ 11 template classes for SIMD vectors

Vector = Matrix x Vector

A **SIMD** vector parallelism **framework**
offered by Grid package

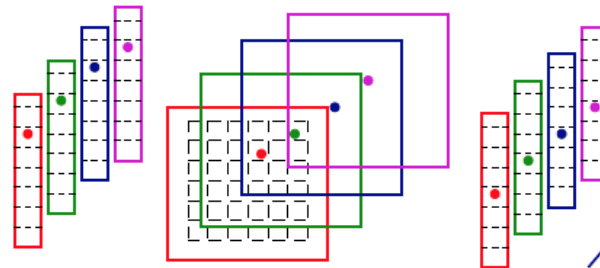
1. The abstraction of vector operations
using modern C++ 11
2. SSE, AVX, AVX2, FMA4, IMCI and AVX512



Reduction of vector sum
is bottleneck for small N



Many vectors = many matrices x many vectors



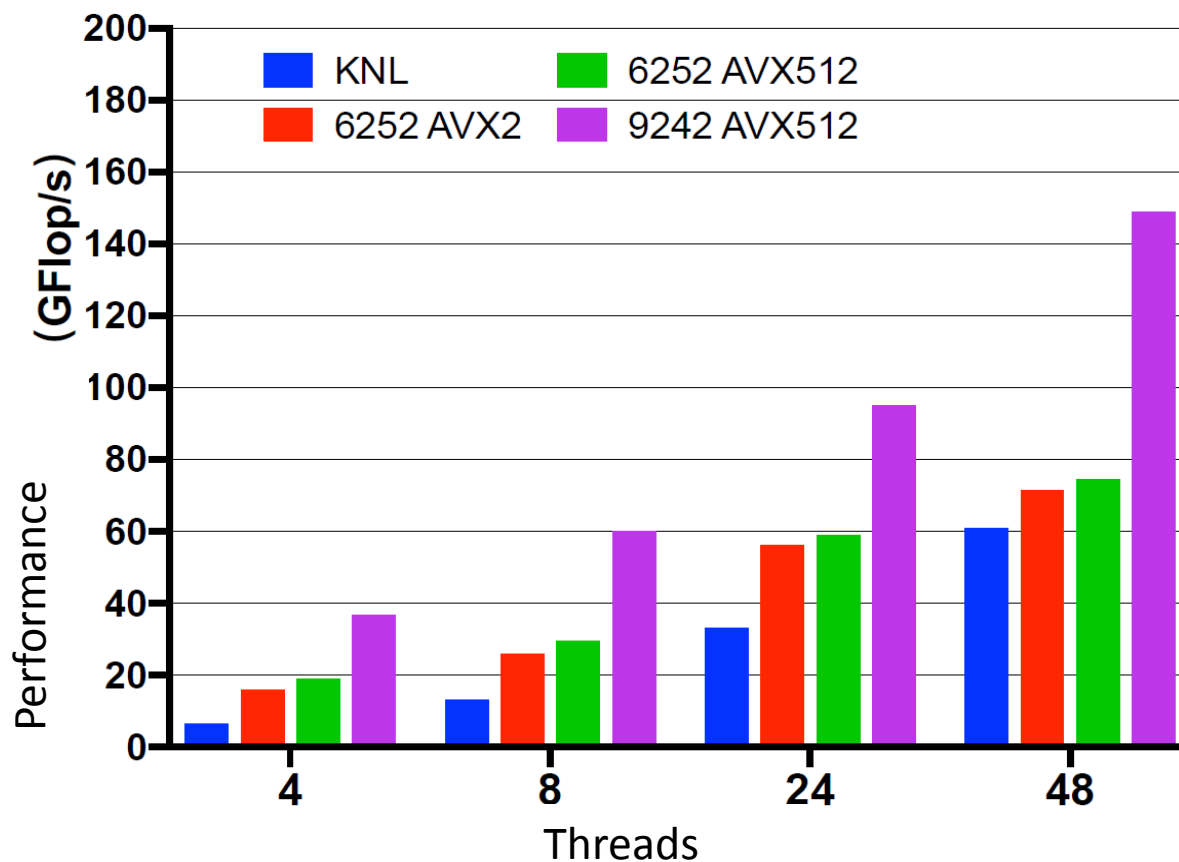
No reduction or SIMD lane
crossing operations.

SIMD interleave

```
#if defined (SSE2)
    typedef __m128 zvec;
#endif
#if defined (AVX1) || defined (AVX2)
    typedef __m256 zvec;
#endif
#if defined (AVX512)
    typedef __m512 zvec;
#endif
class vComplexD {
    zvec v;
    // Define arithmetic operators
    friend inline vComplexD operator + (vComplexD a, vComplexD b);
    friend inline vComplexD operator - (vComplexD a, vComplexD b);
    friend inline vComplexD operator * (vComplexD a, vComplexD b);
    friend inline vComplexD operator / (vComplexD a, vComplexD b);
    static int Nsimd(void);
}
```

Ref.: *Grid: A next generation data parallel C++ QCD library.* Boyle, Peter & Cossu, Guido & Yamaguchi, Azusa & Portelli, Antonin. (2016).

Performance results of Grid



KNL: Intel Xeon Phi 7210
6252: Intel Xeon Gold 6252
9242: Intel Xeon Platinum 9242

Grid tests of vectorization on different processors

BEHIN

SU(3) Vectorization multiplication

for i in (a_1, a_2, a_3)

tmp1 = simd_load(i,i,i)

for j in (α, γ, μ)

tmp2 = simd_load(j_1, j_2, j_3, j_4)

result_re += simd_mul(tmp1,tmp2)

for j in (β, δ, ν)

tmp2 = simd_load(j_1, j_2, j_3, j_4)

result_im += simd_mul(tmp1,tmp2)

for i in (b_1, b_2, b_3)

for j in (α, γ, μ)

tmp2 = simd_load(j_1, j_2, j_3, j_4)

result_im += simd_mul(tmp1,tmp2)

for j in (β, δ, ν)

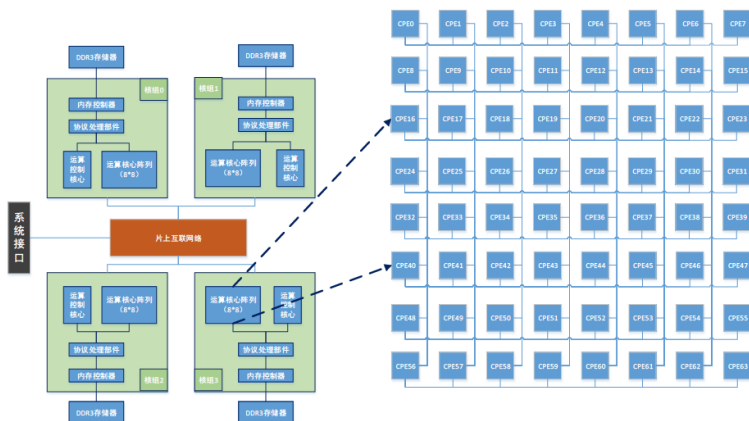
tmp2 = simd_load(j_1, j_2, j_3, j_4)

result_re += simd_mul(tmp1,tmp2)

END

Algorithm of vectorization

1. Partition data by the Z and T axis.
2. 256-bit SIMD vectorization
3. Register communication

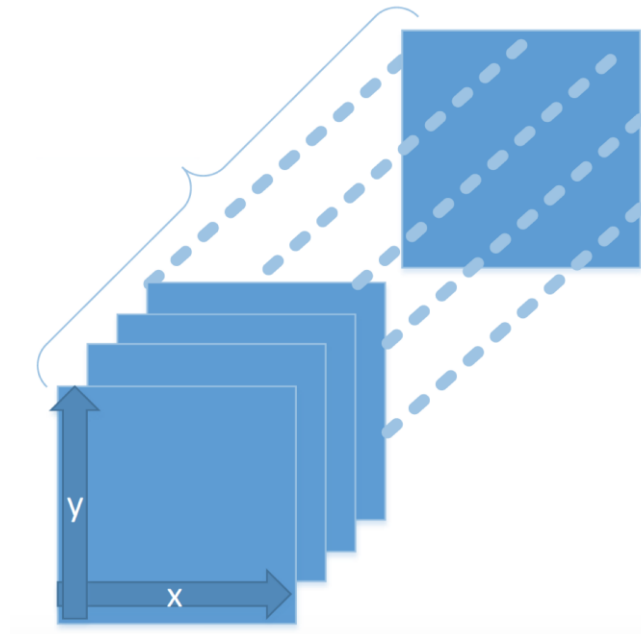


The architecture of Sunway many-core processor

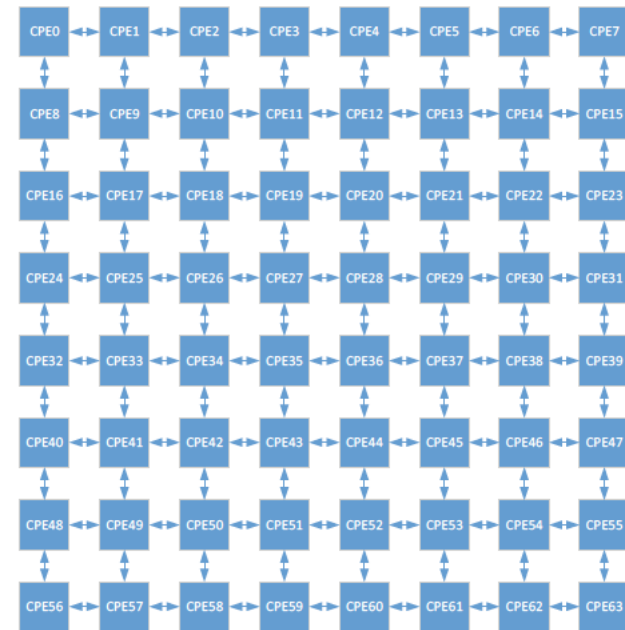
[Sunway TaihuLight Supercomputer](#)

Data organization for vectorization

To fit 64KB size of LDM(Local Data Memory) in
computing processing element (CPE)



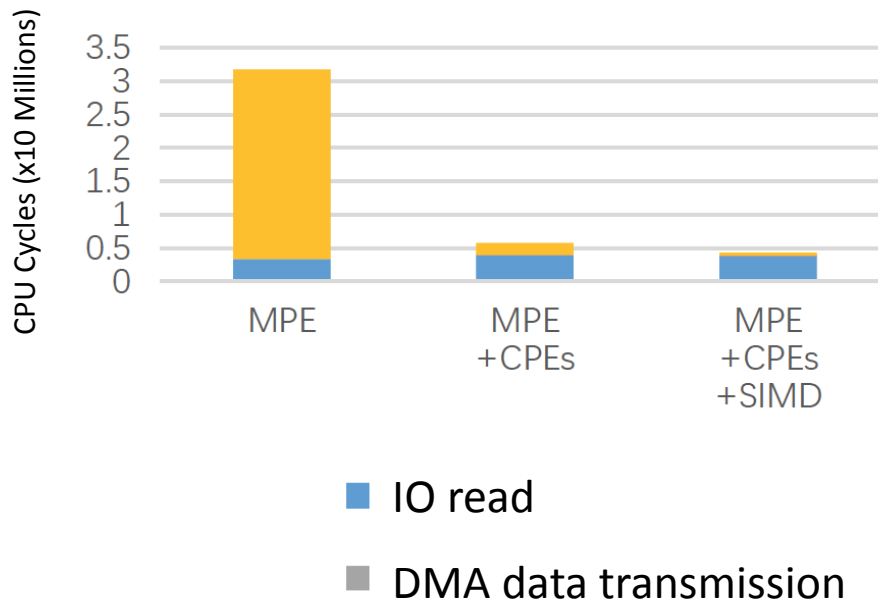
Data segmentation for CPE vectorization



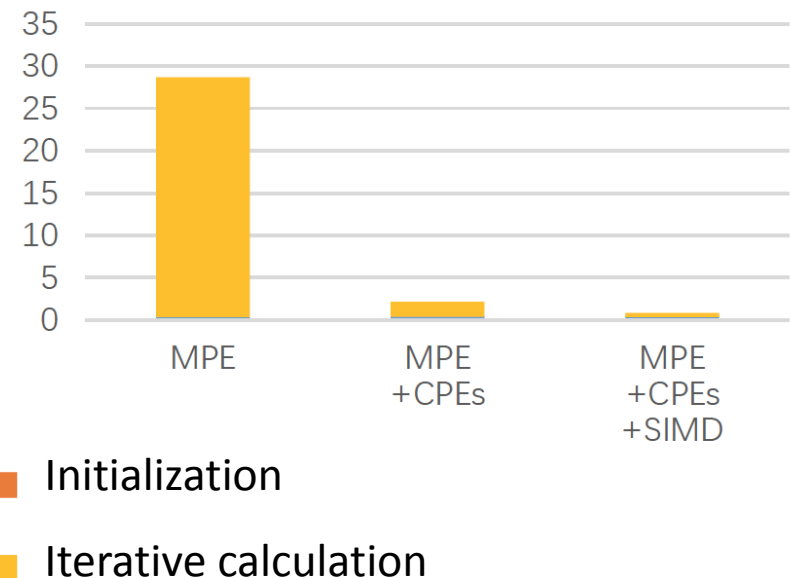
Register communication between CPEs

Performance results

The running time ratio of each part in 10 times iterative calculation



The running time ratio of each part in 100 times iterative calculation



- We are suffering from
 - The poor portability of vectorization code
 - The complexity hardware details overexposed to user
- We are looking forward to
 - A unified programming model with data parallelism
 - Library available on diverse architectures (e.g. SSE, AVX, ASIMD, and NEON).
 - A uniform API for domainal applications

- Lattice QCD is a useful non-perturbative theoretical calculation method, by defining field variables at discrete time-space points.
- High-performance computational methods are required to optimize the data computation and communication in large-scale lattice QCD.
- **My view:** *A Unified data parallel programming model* (e.g. One API by Intel) is expected by lattice QCD application.

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Thank you!

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